The Economics of European Regions: Theory, Empirics, and Policy

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Regression Discontinuity Design -Thistlethwaite and Campbell (1960)

- RDD was introduced by Thistlethwaite and Campbell (1960) as a way of estimating treatment effects in non-experimental setting where the treatment is determined by whether an observed "assignment" variable ("forcing" variable) exceeds a *known* cut-off.
- They use RDD to analyse the impact of merit awards on future academic outcomes.
- They use the fact tat the **allocation** of awards was based on an **observed** test score.
- Main idea: individuals with scores **just below** the cut-off (who did no receive the award) were good comparisons to those **just above** the cut-off (who did receive the award).

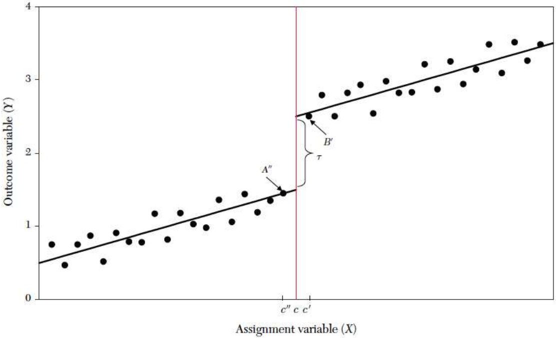
RDD -Thistlethwaite and Campbell (1960) (cont.)

- This assignment generates a sharp *discontinuity* in the treatment (receiving the award) as a *function* of the test score.
- At the same time, there are no reasons, other than the merit award, for future academic outcomes to be a discontinuous function of the test score.

  $\Rightarrow$ the discontinuity jump in the outcome at the cut-off is the *causal effect* of the merit award.

Example Linear RD setup

Figure 1. Lee and Lemieux (2010).



• B' reasonable guess for Y of an individual scoring c (receiving the treatment).
• A'' reasonable guess for Y for the same individual in the counterfactual (not receiving the treatment).
$\Rightarrow B' - A''$ causal estimate.

Example Linear RD setup (cont.)

- In order of the RDD approach to work "all other factors" determining Y must be evolving "smoothly" with respect to X.
- In order to produce a reasonable guess for the treated and untreated states $X = c$ with finite data, one has to use data *away* from the discontinuity
  $\Rightarrow$ the estimate will be dependent on the chosen *functional form*.

Sharp RDD

- In **Sharp RDD** designs the treatment status is a **deterministic** and **discontinuous** function of a covariate X_i.

$$\begin{cases} D_i = 1 & \text{if } X_i \geq c \\ D_i = 0 & \text{if } X_i < c \end{cases}$$

where c is a **known** threshold or cut-off.

- Once we know X_i we know D_i .
- Imbens and Lemieux (2008): there is no value of X_i at which you observe both treatment and control observations.

RDD in potential outcome framework

- Two potential outcomes $Y_i(1), Y_i(0)$ so that the causal effect is $Y_i(1) - Y_i(0)$.
- Fundamental problem of causal inference $\Rightarrow$ focus on average treatment effect $E\left[Y_i(1) - Y_i(0)\right]$.

- In RDD two underlying relationship between average outcome and X: $E\left[Y_i(1)|X\right]$ and $E\left[Y_i(0)|X\right]$.
- All individuals to the right of the cut-off are exposed to treatment and all those to the left are denied to treatment.
- We only observe $E\left[Y_i(1)|X\right]$ to the right of the cut-off and $E\left[Y_i(0)|X\right]$ to the left.
  $\Rightarrow E\left[Y_i(1) - Y_i(0)|X = c\right]$ is the average treatment effect!

RDD in potential outcome framework (cont.)

- Suppose that in addition potential outcomes can be described by a linear, constant effects model:

$$E[Y_i(0)|X_i] = \alpha + \beta X_i$$
$$Y_i(1) = Y_i(0) + \tau$$

- This leads to the regression:

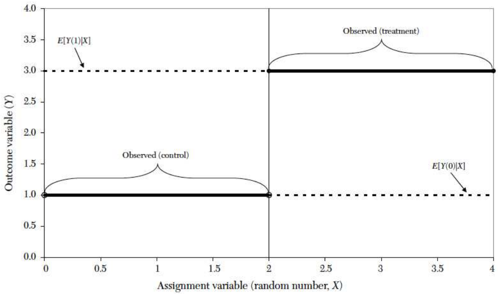
$$Y_i = \alpha + \beta X_i + \tau D_i + \epsilon_i$$

- The key difference of this regression is that D_i is not only correlated with X_i but it is a deterministic function of X_i.

RDD as a local randomized experiment

- The randomized experiment can be thought as an RDD where the assignment variable is $X = \nu$, where ν is a randomly generated number, and the cut-off is c.

Figure 3. Lee and Lemieux (2010).



- The assignment now is random and therefore independent of potential outcomes.
- Moreover, the curves $E[Y_i(1)|X]$ and $E[Y_i(0)|X]$ are flat ($\Rightarrow$ continuous at c).
- The average causal effect is the difference in the mean value of Y just above and just below c.

Key Identifying Assumption

- Key identifying assumption:
  $E[Y_i(1)|X]$ and $E[Y_i(0)|X]$ are continuous in X_i at c.
- This means that all other unobserved determinants of Y are continuously related to the forcing X.
- This allows us to us average outcomes of units just below the cut-off as a **valid counterfactual** for units right above the cut-off variable.
- This assumption cannot be directly tested. But there are some tests which give suggestive evidence whether the assumption is satisfied.

Identification and interpretation - Lee and Lemieux (2010)

1. How do I know whether an RDD is appropriate for my context? When are the identification assumptions plausible or implausible?

   "When there is a continuously distributed stochastic error component to the assignment variable - which can occur when optimizing agents **do not have precise control over the assignment** variable - then the variation in the treatment will be as good as randomized in a neighbourhood around the discontinuity threshold."

   • If individuals have a great control over the assignment variable we can expect that individuals on one side of the threshold to be *systematically* different from those on the other side.
   • But individual will not always be able to have *precise* control.
   • Precise sorting around the threshold is self-selection!

Identification and interpretation - Lee and Lemieux (2010)

2. Is there any way I can test those assumptions?

"Yes. As in a randomized experiment, the distribution of observed baseline covariates should not change discontinuously at the threshold."

• Although is impossible to test this directly, a discontinuity would indicate a **failure** of the identifying assumption.
• As when we want to asses whether the randomized experiment was carried out properly
$\Rightarrow$ the treatment and control groups must be similar in their characteristics.
• If a lagged dependent variable is added as regressor which is pre-determined the local randomization result will imply that the lagged dependent variable will have a continuous relationship with X.

Identification and interpretation - Lee and Lemieux (2010)

3. To what extent are results from RDD generalizable?

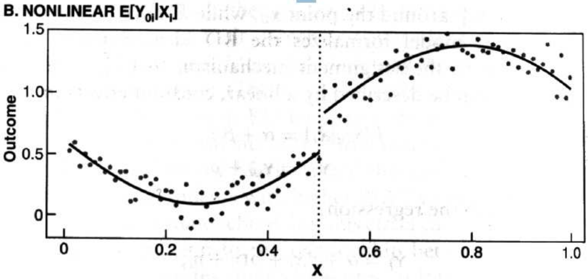
   "The RD estimand can be interpreted as a *weighted average treatment effect*, where the weights are the relative ex ante probability that the value of an individual's assignment variable will be in the neighbourhood of the threshold."

   • If the weights are relatively similar across individuals RDD estimate is closer to the overall average treatment effect.

Sharp Regression Discontinuity - Nonlinear Case

Sometimes the trend relation $E[Y_i(0)|X]$ is nonlinear.

Figure 6.1.1. Angrist and Pischke (2010).



Sharp Regression Discontinuity - Nonlinear Case (cont.)

- Suppose the nonlinear relationship is$E\left[Y_i(0)|X\right] = f(X_i)$ for some reasonably smooth function $f(X_i)$.
- In that case we can construct RDD estimates by fitting:

$$Y_i = f(X_i) + \tau D_i + \eta_i \tag{1}$$

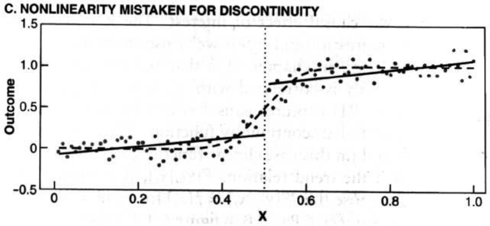
- There are 2 ways of approximating $f(X_i)$:
  - Use a nonparametric kernel method
  - Use a p-th order polynomial: i.e. estimate:

$$Y_i = \alpha + \beta_1 X_i + \beta_2 x_i^2 + \beta_p x_i^p + \tau D_i + \eta_i \tag{2}$$

Internal Validity of RDD Estimates

- The validity of RD estimates depends crucially on the assumption that the polynomials provide an adequate representation of $E\left[Y_i(0)|X\right]$.
- If not what looks like a jump may simply be a non-linearity in $f(X_i)$that the polynomials have not accounted for.

Figure 6.1.1. Angrist and Pischke (2010).



Fuzzy RDD

- The treatment is determined **partly** by whether the assignment variable crosses a cut-off point (imperfect compliance).
- **Fuzzy RD** exploits discontinuities in the **probability** of treatment conditional on a covariate.
- The discontinuity becomes an instrumental variable for treatment status.
- D_i is no longer deterministically related to crossing a threshold but there is a jump in the *probability* of treatment at c.

$$P[D_i = 1|X_i] = \begin{cases} g_1(X_i) & \text{if } X_i \geq c \\ g_0(X_i) & \text{if } X_i < c \end{cases}$$

where $g_1(X_i) \neq g_0(X_i)$.

- $g_1(X_i)$ and $g_0(X_i)$ can be anything as long as they differ at c.

Fuzzy RDD (cont.)

- The relationship between the probability of treatment and X_i can be written as:

$$P\left[D_i = 1|X_i\right] = g_0(X_i) + \left[g_1(X_i) - g_0(X_i)\right] T_i$$

where $T_i = 1(X_i \geq c)$.

- T_i is used as an instrument for D_i.
- The estimated first stage would be:

$$D_i = \gamma_0 + \gamma_1 X_i + \gamma_2 X_i^2 + ... + \gamma_p X_i^p + \pi T_i + \nu_{1i}$$

- The fuzzy RDD reduced form is:

$$Y_i = \mu + \phi_1 X_i + \phi_2 x_i^2 + ... + \phi_p X_i^p + \tau\pi T_i + \nu_{2i}$$

Practical Tips for Estimation

I. Graphical Analysis in RD Designs

II. Estimating the f-Function

III. Testing the Validity of the RD Design

I. Graphical Analysis in RD Designs

1. **Outcome by forcing variable (X_i)**:
   - The standard graph showing the discontinuity in the outcome variable.
   - Construct bins and average the outcome within bins on both sides of the cut-off.
   - Plot the forcing variable X_i on the horizontal axis and the average of Y_i for each bin on the vertical axis.
   - Optionally also plot a relatively flexible regression line on top of the bin means.
   - Inspect whether there is a discontinuity at c.
   - Inspect whether there are other unexpected discontinuities.

   - As robustness for the choice of the bandwidth look at different bin sizes when constructing these graphs (Lee and Lemieux (2010) for details).

I. Graphical Analysis in RD Designs: Outcome by forcing variable

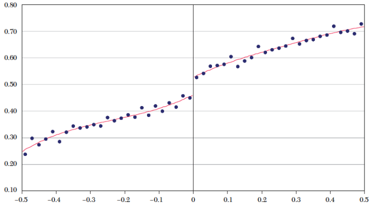


Figure 6. Lee and Lemieux (2010): Bandwidth of 0.02 (50 bins)

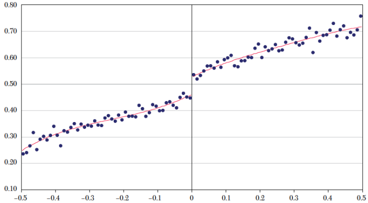


Figure 7. Lee and Lemieux (2010): Bandwidth of 0.01 (100 bins)

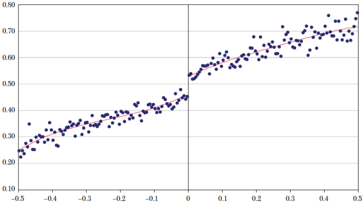


Figure 8. Lee and Lemieux (2010): Bandwidth of 0.005 (200 bins)

I. Graphical Analysis in RD Designs

2. **Probability of treatment by forcing variable if fuzzy RD**.
   - In a fuzzy RD design we also check if the treatment variable jumps at c.
   - If so, there is a first stage!
3. **Covariates by forcing variable**.
   - Construct similar graphs to the one of the outcome but using a covariate as the "outcome".
   - There should be **no jump** in other covariates (e.g., lagged outcome variable).
   - If the covariates would jump at the discontinuity one would doubt the identifying assumption.

I. Graphical Analysis in RD Designs: Covariates by forcing variable

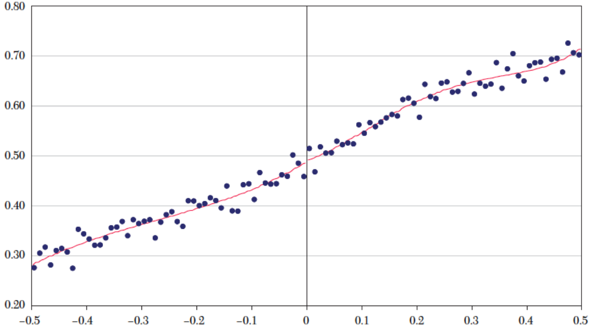


Figure 17. Lee and Lemieux (2010):
Discontinuity in Baseline Covariate (on lagged outcome variable)

I. Graphical Analysis in RD Designs

4. **The density of the forcing variable**.
   - Plot the number of observations in each bin.
   - This plot allows to investigate whether there is a discontinuity in the *distribution* of the forcing variable at the threshold.
   - This would *suggest* that people can *manipulate* the forcing variable around the threshold.
   - This is an indirect test of the identifying assumption that each individual has *imprecise* control over the assignment variable.

I. Graphical Analysis in RD Designs: The density of the forcing variable

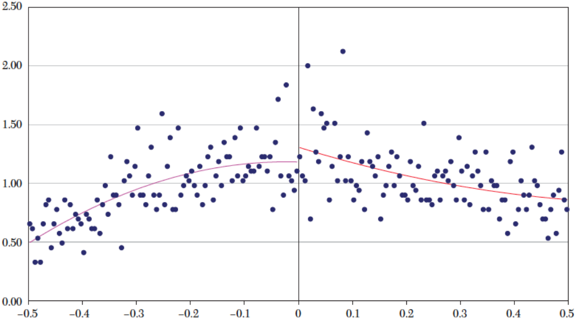


Figure 16. Lee and Lemieux (2010):
Density of the Forcing Variable

II. Estimating the f-Function

- As pointed out before there are essentially two ways of approximating the $f(X_i)$:
  - Kernel regression.
  - Polynomial function.
- There is no right or wrong method. Both have advantages and disadvantages.

II. Estimating the f-Function: the kernel method

- The nonparametric kernel method has its problems in this case because you are trying to estimate regressions at the cut-off point. $\Rightarrow$ "boundary problem":

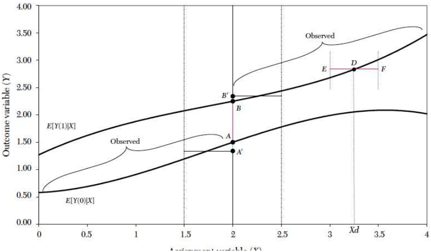


Figure 2. Lee and Lemieux (2010)

- While the "true" effect is AB, with a certain bandwidth a rectangular kernel would estimate the effect as $A'B'$.
- There is therefore systematic bias with the kernel method if the $f(X_i)$ is upwards or downwards sloping.

II. Estimating the f-Function: the kernel method

- The standard solution to this problem is to run local linear regression to reduce the bias.
- The simpler case is the rectangular kernel, which amounts to estimating a standard regression over a window of width h on both sides of the cut-off.
- Other kernel might be chosen but this has little impact in practice.
- While estimating this in a given window of width h around the cut-off is straightforward it is more difficult to choose the bandwidth h.
- See Lee and Lemieux (2010) for two methods to choose the bandwidth (usual trade-off between bias and efficiency).

II. Estimating the f-Function: the polynomial method

- The polynomial method suffers from the problem that uses data far away from the cut-off to estimate the $f(X_i)$ function.
- The equivalent of choosing the right bandwidth for the polynomial method is to use the right order of polynomial.
- See Lee and Lemieux (2010) for a test on the right polynomial.

- Practically:
  - report results for both estimation types;
  - show that including higher order polynomials does not substantially affect the findings;
  - show that the results are not affected by variation in the window around the cut-off.

III. Testing the Validity of the RD Design

1. **Testing the continuity of the density of X**
   - A discontinuity in the density suggests that there is some *manipulation* of X around the threshold.
2. **Explore the sensitivity of the results to the inclusion of baseline covariates**
   - The inclusion of baseline covariates (no matter how they are correlated with outcome) should not affect the estimated discontinuity, if no-manipulation assumption holds.
   - Lee and Lemieux (2010) suggest to simply including the covariates directly, after choosing a suitable order of polynomial
     $\Rightarrow$ significant changes in the estimated effect or increases in the standard errors may be an indication of a misspecified functional form.

References

- Angrist, J. D., and Pischke, J. S. (2008). *Mostly harmless econometrics: An empiricist's companion*. Princeton University Press.
- Becker, S. O., Egger, P. H., and Von Ehrlich, M. (2010). Going NUTS: The effect of EU Structural Funds on regional performance. *Journal of Public Economics*, 94(9), 578-590.
- Lee, D. S., and Lemieux, T. (2010). Regression Discontinuity Designs in Economics, *Journal of Economic Literature*, 48(2), 281-355.
- Pellegrini, G., Terribile, F., Tarola, O., Muccigrosso, T., and Busillo, F. (2013). Measuring the effects of European Regional Policy on economic growth: A regression discontinuity approach. *Papers in Regional Science*, 92(1), 217-233.
- Thistlethwaite, D. L., and Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex post facto experiment. *Journal of Educational Psychology*, 51(6), 309.