The Economics of European Regions: Theory, Empirics, and Policy

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Markovian model of wage convergence

Suppose to represent the dynamics of compensation per employee as follows:

$$\log e_{it} = \phi \log e_{it-\tau} + \gamma_i + \delta_t + \epsilon_{it}, \tag{1}$$

where e_{it} is the compensation per employee in region i at period t, γ_i is a fixed effect for region i, δ_t is a time effect and ϵ_{it} a random component. We assume that $E[\epsilon_{it}] = 0$, $E[\epsilon_{it}^2] = \sigma_\epsilon^2$ and ϵ_{it} is an i.i.d (identical, independent distributed) variable.

Parameter ϕ measures the speed of convergence to $\gamma_i + \delta_t$; in particular, if $\phi = 0$ then we have an instantaneous adjustment, while for $\phi = 1$ there is no any adjustment.

In order to appreciate the similitude with the equation of income convergence used in growth empirics, from Eq. (1) we can derive:

$$\log \frac{e_{it}}{e_{it-\tau}}/\tau \approx g^e_{t-\tau,t} = \frac{\phi - 1}{\tau} \log e_{it-\tau} + \frac{\gamma_i}{\tau} + \frac{\delta_t}{\tau} + \frac{\epsilon_{it}}{\tau}, \tag{2}$$

where $(\phi - 1)/\tau$ is the β parameter of growth equation (for convergence

Markovian model of wage convergence (cont.d)

Defining:

$$\bar{e}_t = \sum_j^R e_{jt}/T \tag{3}$$

the average compensation per employee in our economy composed by R regions and assuming to simply calculation that:

$$\delta_t = (1-\phi)\log \bar{e}_{t-\tau} \tag{4}$$

from Eq. (1), subtracting from both side $\log \bar{e}_t$ and adding and subtracting $\phi \log \bar{e}_{it-\tau}$ in the right hand side, we have:

$$\begin{aligned} \log e_{it} - \log \bar{e}_t &= \phi \log e_{it-\tau} - \phi \log \bar{e}_{t-\tau} + \gamma_i + (1-\phi)\log \bar{e}_{t-\tau} + \\ &+ \phi \log \bar{e}_{t-\tau} - \log \bar{e}_t + \epsilon_{it}, \end{aligned} \tag{5}$$

and therefore:

$$\log \frac{e_{it}}{\bar{e}_t} = \phi \log \frac{e_{it-\tau}}{\bar{e}_{t-\tau}} - log \frac{\bar{e}_t}{\bar{e}_{t-\tau}} + \gamma_i + \epsilon_{it}, \tag{6}$$

Markovian model of wage convergence (cont.d)

In short:

$$\log \tilde{e}_{it} = \phi \log \tilde{e}_{it-\tau} - g^{\bar{e}}_{t-\tau,t} + \gamma_i + \epsilon_{it}, \tag{7}$$

where $\tilde{e}_{it} \equiv e_{it}/\bar{e}_t$ is the relative compensation per employee in region i (relative to regional average) and $\bar{e}_t/e_{it-\tau} \equiv g^{\bar{e}}_{t-\tau,t}$ is the average growth rate of compensation per employee.

Eq. (7) represents a Markovian process. Under the assumption that $g^{\bar{e}}_{t-\tau,t} = \bar{g}^{\bar{e}}$, i.e. growth rate of average compensation per employee is constant over time, it is possible to calculate the expected equilibrium level of compensation per employee of region i, i.e.:

$$E[\log \tilde{e}_i^{EQ}] = \frac{\gamma_i - \bar{g}^{\bar{e}}}{1-\phi}, \tag{8}$$

and the cross-sectional variance of distribution:

$$\sigma^2_{\log \tilde{e}_t} = \phi^2 \sigma^2_{\log \tilde{e}_{t-\tau}} + \sigma^2_\gamma + \sigma^2_\epsilon + 2\sigma_{\tilde{e}_{t-\tau},\gamma}, \tag{9}$$

where $\sigma^2_{\log \tilde{e}_t}$ is the variance of the distribution of $\tilde{e}_t$ and σ^2_γ of γ.

Markovian model of wage convergence (cont.d)

From Eq. (9) we can calculate the cross-sectional variance in equilibrium:

$$\sigma^{2,EQ}_{\log \tilde{e}_t} = \frac{\sigma^2_\gamma + \sigma^2_\epsilon + 2\sigma_{\tilde{e},\gamma}}{1-\phi^2}.. \tag{10}$$

It is also possible to prove that the distribution of $\log \tilde{e}_t$ converge to a normal distribution via standard application of Central Limit Theorem. This implies that Eqq.(8) and (10) provide all information on the equilibrium distribution of $\tilde{e}$

Estimation of the Markovian model of wage convergence

Eq. (7) can be estimated by a simple panel regression with fixed and time effects. An application to regional compensation per employee for EU28 regions can be found in Table 6, where ϕ is estimated equal to 0.74 and overall regression explain 94% of total variance. Other examples are presented in the code available on website.

	Dependent variable:
	log(compensation.tot)
log(lag.compensation.tot)	0.740^{***}
	(0.003)
Observations	3,774
R^2	0.941
Adjusted R^2	0.936
F Statistic	$55{,}877.850^{***}$ (df = 1; 3502)
Note:	*p<0.1; **p<0.05; ***p<0.01

Estimation of the Markovian model of wage convergence

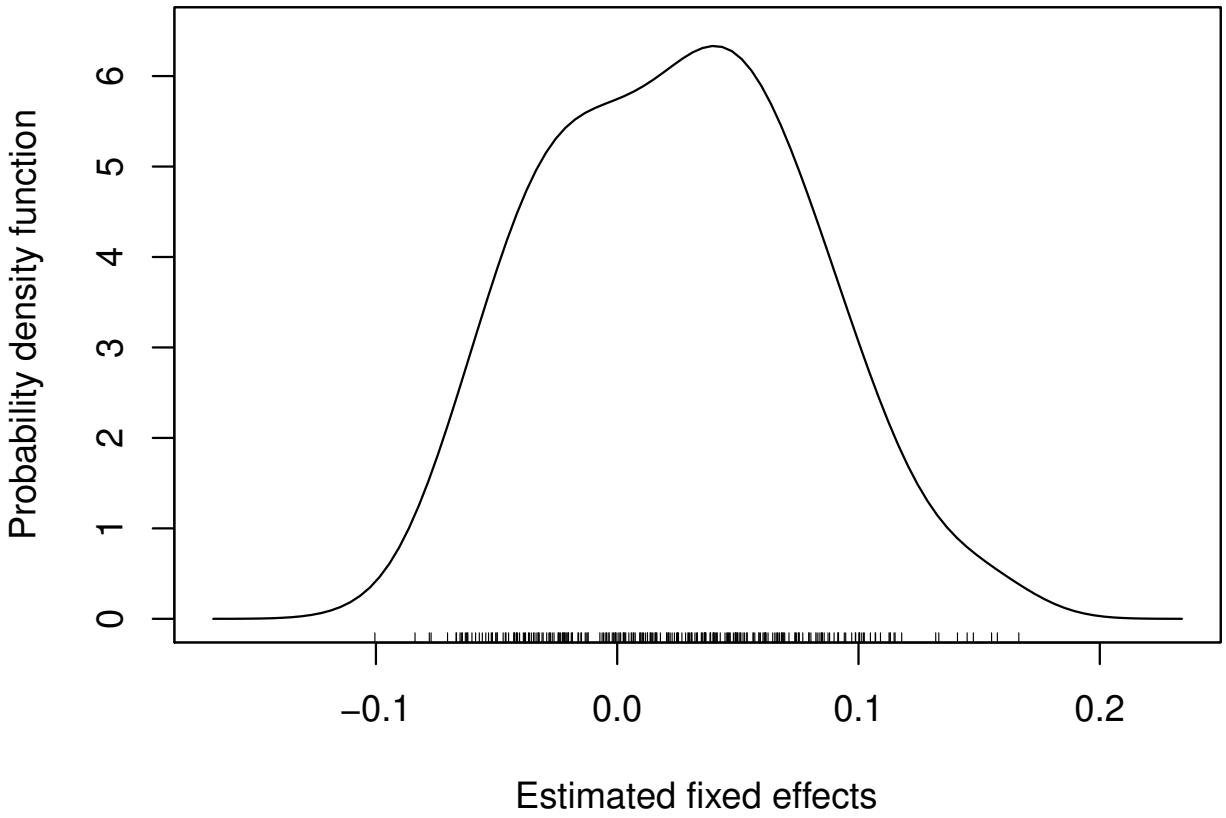

Figura: Estimated fixed effects from the Markovian model of wage convergence